Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **8 (2001)** 35-51

EXISTENCE AND UNIQUENESS RESULTS FOR A CLASS OF NONLOCAL ELLIPTIC AND PARABOLIC PROBLEMS

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Abstract. We study some class of nonlocal problems. In the parabolic homogeneous case we show in particular the possibility of quenching of the solution.

AMS (MOS) subject classification. 35JXX, 35KXX

1. Introduction

It has been shown in [3] that in general solving a nonlinear nonlocal elliptic problem reduces to solve a non linear equation in $\mathbf{R}$ or in a finite dimensional space. This constrasts with the local case where in order to solve a non linear elliptic or parabolic equation one has to rely on the Schauder fixed point theorem in an infinite dimensional space. However, even if theoretically the things are clear it is not always possible to find out the equation to be solved in $\mathbf{R}^n$. One of the goal of this work is to investigate a certain number of situations where this is possible. A model problem could be the following: find a function $u(x,t)$ such that

$$\begin{cases} u_t - a(l(u))\Delta u = f \quad \text{in} \quad \Omega \times (0,T), \\ u(x,t) = 0 \quad \text{on} \quad \Gamma \times (0,T), \\ u(x,0) = u_0(x) \quad \text{in} \quad \Omega. \end{cases} \tag{1.1}$$

Here Ω is a bounded open subset in $\mathbf{R}^n$, $n \geq 1$ with smooth boundary Γ. T is some arbitrary time. a is some function from $\mathbf{R}$ into $(0,+\infty)$ and l a continuous linear form on $H_0^1(\Omega)$ that is to say an element of $H^{-1}(\Omega)$ (see [5], [7] or [8] for the background on Sobolev spaces).

Such an equation arises in various situations. For instance u could describe the density of a population (for instance of bacteria) subject to spreading. The diffusion coefficient a is then supposed to depend on a non local