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CRITICAL SPEED FOR QUENCHING

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Abstract. A one-dimensional heat equation with a nonlinear, concentrated quenching source that moves with constant speed through a diffusive medium is examined. Bounds are established for a critical speed above which quenching will not occur. When quenching does occur, bounds are given for the quenching time. For the special case of a power law nonlinearity, the growth rate near quenching is derived. The analysis is conducted in the context of a nonlinear Volterra integral equation that is equivalent to the initial-boundary value problem.

AMS (MOS) Subject Classifications: 35K55, 35K57, 45D05, 45G05, 45M05.

1 Introduction

In quenching problems the solution remains bounded while the first order time derivative becomes unbounded in finite time. We examine a quenching problem for the heat equation in an infinite one-dimensional strip. The nonlinear, concentrated quenching source term moves with constant speed through the diffusive medium. In the case of a stationary source, it is known that, under appropriate conditions, quenching will occur [2, 3, 4, 5, 7, 10]. The main issue of interest here is to establish that there is a critical speed above which quenching will not take place. A related problem that deals with the critical speed to avoid blow-up in a reactive-diffusive medium has recently been examined in [6].

A stationary quenching source, which is concentrated at a fixed position, will continuously supply heat in an attempt to raise the local temperature to a prescribed level. Under appropriate conditions, this energy supply will be adequate to attain the prescribed temperature level in a finite time, thereby achieving the quenched state. By allowing the source to move, it is perpetually exposed to new surroundings that are relatively cool. Thus, if the source moves at a sufficiently high speed, it may not be able to supply enough energy at any fixed site to achieve quenching.

To gain some insight into this phenomenon, we introduce a one-dimensional model for the temperature of a diffusive medium with a concentrated quenching source that moves at a constant speed. We consider the temperature