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QUENCHING PROBLEM OF A FUNCTIONAL PARABOLIC EQUATION

C.V. Pao

Department of Mathematics
North Carolina State University
Raleigh, North Carolina 27695-8205

Abstract. This paper is concerned with the quenching problem of a parabolic equation in a multidimensional domain Ω_α ($\alpha > 0$ is a parameter) where the singular reaction function depends on both the unknown function u and a functional value $K * u$. It is shown that there exists a critical value α^* such that a unique global solution of the time-dependent problem exists and converges to a positive steady-state solution if $\alpha < \alpha^*$, and the solution quenches if $\alpha > \alpha^*$. Also discussed are the existence and nonexistence of positive steady-state solutions and the convergence of the time-dependent solution to the minimal steady-state solution.
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1 Introduction

Since the work of [11] by Karawada in 1975 for a one-dimensional heat equation the problem of quenching for parabolic partial differential equations has been given considerable attention, and various types of differential operators and reaction functions have been treated in the current literature (e.g. see [1, 2, 4-14]). In this paper we study the quenching problem for a class of functional parabolic boundary-value problems in the form

$$\begin{aligned} &u_t - Lu = f(u, K * u) && \texttt{in} \quad (0,\infty)\times\Omega_\alpha \\ &u(t,x) = 0 && \texttt{on} \quad (0,\infty)\times\partial\Omega_\alpha \\ &u(0,x) = u_0(x) && \texttt{in} \quad \Omega_\alpha\ , \end{aligned} \tag{1.1}$$

where for each $\alpha > 0$, Ω_α is a smooth bounded domain in $\mathbf{R^n}$ with boundary $\partial\Omega_\alpha$, L is a uniformlly elliptic operator in the self-adjoint form

$$Lu = \sum_{i,\,j=1}^{n} \frac{\partial}{\partial x_j}(a_{ij}(x)\frac{\partial u}{\partial x_i})$$