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MONOTONE ITERATIVE AND FINITE ELEMENT METHODS ON REACTION-DIFFUSION EQUATIONS

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Abstract. A new algorithm is developed to solve the time dependent nonlinear reaction diffusion equation. We discretize the reaction diffusion equation by the Galerkin finite element method, permute an elliptic equation over time using backward Euler discretization then solve this system by a monotone iterative method. We are able to effectively blend the strengths of the finite element and monotone iterative methods into a stable algorithm. Uniqueness, convergence, and stability are then discussed.

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1 Introduction

Reaction-diffusion equations are widely used in biology, ecology, biochemistry and engineering sciences. A fruitful method for the treatment of these equations, both analytically and numerically, is the method of upper and lower solutions and its associated monotone iterative method[8]. There has been extensive work done with numerical solutions of Reaction-diffusion equations using the finite difference and monotone iterative methods[5, 6, 7]. The main drawback of this approach is clearly what is the type of shortcoming associated with the finite difference method compared with other methods, such as the finite element method. The Finite Element method is perhaps the most robust of the techniques available for finding numerical solutions. The major advantage of using finite elements over finite differences is the relative ease with which the boundary conditions are handled. Irregularly shaped boundaries are quite difficult to handle using finite differences as each boundary condition with a derivative must be approximated by a difference quotient at the grid points, and irregular shaping of the boundary makes placing the