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# A FAST AND HIGHLY ACCURATE NUMERICAL METHOD FOR THE EVALUATION OF AMERICAN OPTIONS

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**Abstract.** We introduce a novel practical approach to the valuation of American options which involves the exact reformulation of the problems and numerical solutions over very small regions. Numerical examples and analysis show that our algorithm leads to very rapid and accurate results.

**Keywords.** American option, variational inequality, integral boundary condition, finite element method, error estimate.

**AMS (MOS) subject classification:** 90A09, 65K10, 65M12, 65M60

## 1 Introduction

The classic Black-Scholes equation reduces the question of determining the optimal exercise price of an American call or put option to that of finding the free boundary for a partial differential inequality. It is common practice to remove some of the mathematical complexity involved by introducing a change of variable. As may be expected, this procedure leads to other difficulties, since the new problem needs to be solved over an infinite range. In practice, this is dealt with by numerically solving the inequality over a large but finite range. In this way, two difficulties are introduced: (a) the computer simulations must be run over a "large" region and thus are relatively slow; (b) an artificial boundary value must be imposed, thus effecting the accuracy of the simulation.

In this paper we eliminate both difficulties by the introduction of a new nonlocal boundary condition, which is mathematically exact and which enables us to restrict the calculations needed for the solution of the problem to a very narrow region. We illustrate the quality and speed of our results by explicit comparison with results obtained by other authors. We implement a finite element code, but we emphasize that the other numerical approaches commonly employed (finite difference method, finite volume method, etc.) can benefit in speed and accuracy from our considerations. The analytical and mathematical details of the procedures we present are somehow involved,