Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **8 (2001)** 149-159

SECOND ORDER STURM-LIOUVILLE BVP'S WITH IMPULSES AT VARIABLE MOMENTS[1]

M. Frigon[1] and D. O'Regan[2]

[1]Département de Mathématiques et Statistique
Université de Montréal
C. P. 6128, Succ. Centre-ville, Montréal, H3C 3J7, Canada
[2]Department of Mathematics
National University of Ireland, Galway, Ireland
frigon@dms.umontreal.ca and donal.oregan@nuigalway.ie

Abstract. In this paper, existence results are obtained for second order Sturm-Liouville BVP's with impulses at variable times. The proofs rely on a fixed point theorem for composition of acyclic maps.

AMS (MOS) subject claassifications:34A37, 34B15

1. Introduction

In this paper, the second order problem with impulses at variable times

$$\text{(P1)} \qquad \begin{aligned} & x''(t) = f(t, x(t), x'(t)), && \text{a.e. } t \in [0,1], \\ & x(t^+) = I(x(t)), && \text{if } t = \tau(x(t)), \\ & x'(t^+) = J(x(t)), && \text{if } t = \tau(x(t)), \end{aligned}$$

with the Sturm-Liouville boundary condition

$$\text{(SL)} \qquad \begin{aligned} & x(0) - ax'(0) = \alpha, \quad a \geq 0, \\ & x(1) + bx'(1) = \beta, \quad b \geq 0, \end{aligned}$$

is studied.

The literature on second order impulsive boundary value problems is devoted totally to the case where the impulses are at fixed times (i.e. when τ is constant), see for example [3]–[5]. This is in contrast to first order initial value problems where some results were obtained with impulses at variable times, see [2], [6], [7], [10].

To our knowledge, it is the first paper to treat second order boundary value problems with impulses at variable times.

For sake of simplicity, in section 2, we begin by discussing the problem (P1), (SL) with homogenous Dirichlet boundary condition (i.e. $a = b = \alpha = \beta = 0$). To obtain the existence of a solution, we use fixed point theory for composition of acyclic maps [8], [9], and the fact that the solution set of an initial value problem has R_δ–values [1], [11]. Some of the ideas of this

[1]This work was partially supported by CRSNG Canada and INTAS-96-0915