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BOUNDED SOLUTIONS OF A NON-AUTONOMOUS LOGISTIC EQUATION WITH FINITE DELAY

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Abstract. A necessary and sufficient condition for the existence of a bounded and uniformly positive solution of a logistic equation with finite delay is obtained. Also, sufficient conditions for global asymptotic stability are given.

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1 Introduction

Consider the scalar non-autonomous logistic equation with finite delay

$$x' = xf(t, x_t), \quad x \geq 0 \qquad (*)$$

where f is a monotone decreasing functional. In this paper we obtain a necessary and sufficient condition for the existence of a bounded and uniformly positive solution of (*) defined on $\mathbb{R}$. Also, some sufficient conditions for global asymptotic stability will be given.

The existence of bounded and positive solutions on $\mathbb{R}$ of a logistic ordinary differential equation has been broadly studied, in recent literature, see for instance [11]. With reference to a logistic equation with delay we will next mention the work of Seifert [10], that studies a type of non-autonomous logistic equation with infinity delay and obtains sufficient conditions under which the equation has a positive, almost periodic solution on $\mathbb{R}$, which attracts all other positive solutions as $t \to +\infty$.

In order to simplify, we will consider equations with finite delay. Our characterization of boundedness for (*) is motivated by results referred to ordinary differential equations by Ortega in [7] and by Ortega and Tineo in [8]. The same ideas could be applied to some logistic equations with infinity delay, like Seifert [10].

In contrast to the situation for the o.d.e., for the delay equations, the existence of a bounded positive solution does not guarantee the global asymptotic stability. In fact, it is known that there are simple autonomous equations having an equilibrium and also non trivial periodic solutions (See Seifert [9]). In our situation we will obtain sufficient conditions for global stability when the equation is asymptotically autonomous and also for a class of equations that tends to an ordinary differential equation as $t \to +\infty$.