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# CONE-VALUED LYAPUNOV FUNCTIONS AND STABILITY OF HYBRID SYSTEMS

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**Abstract.** We establish stability and practical stability in two measures for hybrid impulsive systems by the cone-valued Lyapunov function method. We deduce results for the scalar and vector Lyapunov function method as special cases.

## 1 Introduction

In [13] the concept of impulsive hybrid systems was introduced and the method of scalar Lyapunov functions was used, along with the comparison principle to analyse and develop stability criteria for such systems. A general description of hybrid systems is contained in [13]. Throughout this paper we shall assume such and adopt the notations contained therein. The method of cone-valued Lyapunov functions is now known ([1, 2, 3, 4, 5, 6, 9, 11, 12]) to be beneficial in applications and circumvents the limitations of the useful and well-known method of both the scalar and vector Lyapunov functions. In this paper, we shall employ the method of cone-valued Lyapunov functions to investigate the stability of impulsive hybrid systems. We first establish new comparison results in terms of cone valued Lyapunov functions which will be used to study stability criteria for hybrid impulsive systems. We also deduce as special cases of our results, the extensions of results of [13] to vector-valued Lyapunov functions.

## 2 Preliminaries

We consider the impulsive system

$$\begin{aligned} x' &= f(t, x, \lambda_k(x_k)), t \in [t_k, t_{k+1}] \\ x(t_k^+) &= x_k^+, x_k^+ = x_k + I_k(x_k), k = 0, 1, 2, .... \\ x_k &= x(t_k), I_0(x_0) = x_0, x(t_0^+) = x_0 \end{aligned} \tag{1}$$