Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **8 (2001)** 215-226

EXISTENCE OF POSITIVE SOLUTIONS FOR A CLASS OF SINGULAR SUBLINEAR BOUNDARY VALUE PROBLEMS[1]

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Abstract. This paper investigates the existence of positive solutions for a class of singular sublinear boundary value problems. A necessary and sufficient conditon for the existence of $C[0,1]$ positve solutions as well as $C^1[0,1]$ positive solutions is given by using the method of lower and upper solutions with the fixed point theorems.
AMS(MOS) Subject Classification: 34B15

1 Introduction

Consider the singular boundary value problems of second order ordinary differential equations

$$x'' + f(t,x) = 0, \qquad t \in (0,1), \tag{1}$$

$$ax(0) - bx'(0) = 0, \qquad cx(1) + dx'(1) = 0, \tag{2}$$

where $a \geq 0, b \geq 0,\ c \geq 0,\ d \geq 0,\ \rho = ac + ad + bc > 0$. By singularity we mean that the function f in (1) is allowed to be unbounded at the end points $t = 0$ and $t = 1$. Recently such problems have been studied by many authors (see [1-9] and their references). In the special case where $a = c = 1,\ b = d = 0$, the positive solutions of SBVP (1) and (2) have been discussed by means of the method of lower and upper solutions in [8]. When $f(t,x) = p(t)x^{\alpha},\ p(t) \in C(0,1),\ p(t) > 0$, the existence and uniqueness of positive solutions for the special cases (i): $b = d = 0,\ \alpha < 0$ and (ii): $b = d = 0,\ 0 < \alpha < 1$ have been studied completely by Taliaferro in [1] with the shooting method and by Zhang in [7] with the method of lower and upper solutions, respectively. Now, in this paper, we shall give a necessary and sufficient conditon for the existence of $C[0,1]$ positve solutions as well as $C^1[0,1]$ positive solutions of the singular problem (1),(2) by using the method of lower and upper solutions with the fixed point theorems.

[1]Research supported by NNSF of China 19771053 and Shandong Province, and the Doctor Foundation of Shandong Institute of Architectural and Engineering