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SOME MULTIPLICITY RESULTS FOR PERIODIC SOLUTIONS OF A RAYLEIGH DIFFERENTIAL EQUATION

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Abstract. We study the existence and multiplicity of T-periodic solutions for a Rayleigh equation, with conditions on the nonlinearity that include some classical situations.
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1. Introduction

In this paper, we are concerned with the periodic problem for a Rayleigh equation

$$\begin{array}{c} x'' + f(x') + g(t,x,x') = \bar{p}, \\ x(0) = x(T),\ x'(0) = x'(T). \end{array} \tag{1}$$

Our aim is to study the structure of the set of "mean values" $\bar{p}$ for which there exists at least one or at least two solutions of problem (1). Such sets turn out to be intervals which can be bounded or unbounded.

The literature contains a large variety of such results a example of which is the classical Ambrosetti-Prodi problem [1], see also [6], [4] and the references therein. Other related situations concern periodic nonlinearities (see for instance [10], [8]) or in case of a Dirichlet problem, nonlinearities depending only on the derivative [7]. Our main concern was to study similar situations for periodic solutions of the Rayleigh equation (1) which seems to be little studied. We were also concerned with extension to equations which satisfy Carathéodory conditions. This forced us to write in Section 2 a theorem which relates the existence of ordered strict upper and lower solutions with the degree of a suitable operator. Such a result in the framework of Carathéodory conditions does not seem to be classical.

In Section 3, we consider restoring forces $g(t,x,x')$ uniformly bounded by L^2-functions. This includes periodic nonlinearities such as in the pendulum equation. In this case, the set of admissible "mean values" $\bar{p}$ turns out to be an interval I, which we can estimate for the damped pendulum equation. If $g(t,x,x')$ is periodic in x, we prove existence of two solutions in the interior of I. The proof of this result is based on the existence of strict upper and lower solutions. This forced us to impose some uniform continuity condition