Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **8 (2001)** 383-399

ON THE ASYMPTOTIC BEHAVIOR OF SOME NONLINEAR DELAY DIFFERENTIAL SYSTEMS

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Abstract. In this paper persistence, nonnegativeness and asymptotic decay towards the trivial solution are investigated for all the solutions of a class of nonlinear delay differential systems related to population dynamics. A general existence theorem for delay differential systems with boundary conditions in noncompact intervals is also presented.
AMS (MOS) subject classification:34K20, 34K25, 92D25

1 Introduction

In this paper we consider a nonlinear delay differential system of the form

$$\begin{aligned} \dot{x}_1 &= -a_1(t)x_1 - c_1(t)g_1(x_1, x_2) + b_1(t)f_1(x_1(t-r), x_2(t-r)) \\ \dot{x}_2 &= -a_2(t)x_2 - c_2(t)g_2(x_1, x_2) + b_2(t)f_2(x_1(t-r), x_2(t-r)) \end{aligned} \tag{1}$$

where the delay r is assumed to be a given positive constant and the following set of assumptions are supposed:

1. a_i, b_i, c_i, $i = 1, 2$, are nonnegative continuous functions defined on the real halfline $[0, \infty)$,

2. g_i, $i = 1, 2$, is a nonnegative, locally Lipschitz continuous function defined in the set $[0, \infty) \times [0, \infty)$, such that $g_i(0, 0) = 0$; in addition the functions $g_1(x_1, x_2)/x_1$ and $g_2(x_1, x_2)/x_2$ are upper bounded in every compact $Q = [0, L] \times [0, L]$, that is, for every $L > 0$ there exist $\ell_1(L)$ and $\ell_2(L)$ such that

$$\frac{g_1(x_1, x_2)}{x_1} < \ell_1(L) \text{ for every } (x_1, x_2) \in [0, L]^2 \tag{2}$$

$$\frac{g_2(x_1, x_2)}{x_2} < \ell_2(L) \text{ for every } (x_1, x_2) \in [0, L]^2 \tag{3}$$

3. f_i, $i = 1, 2$, is a nonnegative, continuous function defined in $[0, \infty) \times [0, \infty)$, such that $f_i(0, 0) = 0$, and

$$f_i(a, \cdot) \le a \quad \forall a \in [0, \infty), \qquad f_i(\cdot, b) \le b \quad \forall b \in [0, \infty). \tag{4}$$