Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **8 (2001)** 415-423

GLOBAL EXISTENCE AND BLOW-UP FOR A SYSTEM OF WAVE EQUATIONS COUPLED IN THE BOUNDARY CONDITIONS

Azmy S. Ackleh and Keng Deng

Department of Mathematics
University of Louisiana at Lafayette
Lafayette, Louisiana 70504

Abstract. We study the initial-boundary value problem

$$\begin{array}{lll} u_{tt}=u_{xx}, & v_{tt}=v_{xx}, & 0<x<\infty, \quad t>0, \\ -u_x(0,t)=|v(0,t)|^p, & -v_x(0,t)=|u(0,t)|^q, & t>0, \\ u(x,0)=f(x), & v(x,0)=h(x), & \\ u_t(x,0)=g(x), & v_t(x,0)=k(x), & 0<x<\infty. \end{array}$$

We establish criteria for global existence and blow-up of solutions, and we present the growth rates at blow-up.

Keywords. System of wave equations, global existence, blow-up.
AMS (MOS) subject classification: 35B40, 35L05, 35L55.

1 Introduction

In this paper, we consider the following initial-boundary value problem:

$$\begin{array}{lll} u_{tt}=u_{xx}, & v_{tt}=v_{xx}, & 0<x<\infty, \quad t>0, \\ -u_x(0,t)=|v(0,t)|^p, & -v_x(0,t)=|u(0,t)|^q, & t>0, \\ u(x,0)=f(x), & v(x,0)=h(x), & \\ u_t(x,0)=g(x), & v_t(x,0)=k(x), & 0<x<\infty. \end{array} \tag{1.1}$$

Here $0<p,q<\infty$, and all of the initial values are continuous. In order to motivate the main results for problem (1.1), we recall some old results for two related problems. On the one hand, in [3, 4] Del Santo et al. and Deng studied the initial value problem

$$\begin{array}{lll} u_{tt}=u_{xx}+|v|^p, & v_{tt}=v_{xx}+|u|^q, & -\infty<x<\infty, \quad t>0, \\ u(x,0)=f(x), & v(x,0)=h(x), & \\ u_t(x,0)=g(x), & v_t(x,0)=k(x), & -\infty<x<\infty. \end{array} \tag{1.2}$$

They proved that if $1<pq<\infty$, every nontrivial solution of (1.2) blows up in finite time. On the other hand, for the initial-boundary value problem

$$\begin{array}{lll} u_t=u_{xx}, & v_t=v_{xx}, & 0<x<\infty, \quad t>0, \\ -u_x(0,t)=|v(0,t)|^p, & -v_x(0,t)=|u(0,t)|^q, & t>0, \\ u(x,0)=f(x), & v(x,0)=h(x), & 0<x<\infty, \end{array} \tag{1.3}$$