GENERAL FORMS OF THE SIMPLEST NORMAL FORMS OF BOGDANOV-TAKENS SINGULARITIES

Jing Li[1],Duo Wang[2] and Wei Zhang[3]

[1] College of Applied Mathematics and Physics
Beijing Polytechnic University, Beijing 100022,P.R.China
Email address:leejing@bjpu.edu.cn
[2]School of Mathematical Science
Peking University, Beijing 100871, P.R.China
Email:dwang@math.pku.edu.cn
[3]College of Mechanical Engineering,
Beijing Polytechnic University, Beijing 100022, P.R.China
Email address:sandyzhang0@yahoo.com

Abstract. In this article the method of transformation with parameters is applied to compute the simplest normal forms of the Bogdanov-Takens singularities. Under some different conditions, general forms of the simplest normal forms up to 12th degree are obtained for several cases.
Keywords. Normal form, Lie algebra, Transformation, Bogdanov-Takens singularities.
AMS (MOS) subject classification:34c20, 58f36

1 Introduction

Recently further simplification of normal forms of ordinary differential equations has been widely noticed by the researchers since the classical normal form theory may not give the simplest normal forms with respect to near identity transformations and coefficients of classical normal forms are generally not uniquely determined by the original equations even if the form of the normal form is fixed, see for example, [6] and references therein.

Ushiki [7] first developed the method of multiple Lie brackets involving nonlinear terms to obtain further reduction of the classical normal forms. He gave the simplest normal forms up to 4th degree for the Bogdanov-Takens singularities. Baider and Sanders [1] [2] investigated the uniqueness of formal normal forms with the aid of new grading functions. The advantage of their method is that the lowest homogeneous terms in the sense of new grading include not only linear terms but also some nonlinear terms in the sense of classical grading so that one may use fewer Lie brackets to get the simplest normal forms. In [2] Baider and Sanders gave the unique (formal) normal forms of Bogdanov-Takens singularities for some cases. But for the case $\mu = 2\nu$(see [2] for the details), they gave only some basic computation and the idea of how to get the unique normal form. Kokubu, Oka and Wang [6] defined nth order normal forms by using n Lie brackets. They proved that the infinite