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ON THE NORMAL FORM OF THE TRIPLE–ZERO DEGENERACY WITH GEOMETRIC MULTIPLICITY TWO

E. Gamero

Department of Applied Mathematics II,
University of Seville
41092-Sevilla, Spain

Abstract. This paper deals with three-dimensional autonomous systems, having a linear degeneracy corresponding to a triple-zero eigenvalue with two Jordan blocks. More specifically, a computational approach to obtain normal forms for this degeneracy is presented. The method allows to get explicit expressions for the normal form coefficients, and to analyze some additional simplifications that can be achieved. The results are applied in some particular situations: suspended Takens-Bogdanov singularity, and the presence of symmetry. We also present some results on bifurcations of a five-parameter unfolding of the normal form. The idea is to perform a blow-up by a singular rescaling, leading to a reversible volume-preserving system, whose analysis will provide useful information for the unfolding.

Keywords. Normal forms, triple-zero linear degeneracy, global bifurcations.
AMS (MOS) subject classification: 34C, 37G.

1 Introduction

In the analysis of a dynamical or bifurcation problem, one usually tries to reduce it to another one which is equivalent and easier to study. One of the main tools in this subject is the normal form theory. It is well known that, when one is dealing with a hyperbolic equilibrium, the local phase portrait can be determined from the linear part, applying Hartman's theorem. However, when the equilibrium is nonhyperbolic, the full consideration of the nonlinear terms in the system is required. The normal form theorem determines the nonlinear terms that can be removed by means of successive changes of variables, providing simpler equivalent systems which give account of the original dynamics.

In this paper, following a line of previous works (see Algaba et al. [2] and references therein) we present a computational approach to obtain normal forms for autonomous systems with an equilibrium point having a triple-zero eigenvalue with geometric multiplicity two. This corresponds to the coupling of two Jordan blocks, one corresponding to a Takens-Bogdanov singularity and the other one to a simple-zero eigenvalue.