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THE NORMAL FORM FOR A HOPF/STEADY-STATE MODE INTERACTION ON A HEXAGONAL LATTICE

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Abstract. The interaction of three-dimensional steady and oscillatory patterns on a hexagonal planar lattice is considered, when the ratio of the pattern lengthscales is $1:\sqrt{3}$. The normal form for the mode interaction is derived from symmetry considerations for the simplest case; this is where the size of the imposed lattice is chosen to ensure that the relevant symmetry group, $D_6 \ltimes T^2$, acts by its fundamental representation on both the steady and oscillatory modes. This analysis is of interest for many pattern-forming systems because the wavenumbers involved in mode interactions of this kind are those selected naturally by the system in a spatially-extended domain.

Keywords: Bifurcation theory, symmetry, mode interaction

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1 Introduction

A common feature of many spatially-extended continuum systems is that they undergo pattern-forming instabilities of an initially uniform state. These often produce steady regular periodic patterns, for example stripes or hexagons. In some cases, though, the loss of stability is via a Hopf bifurcation and oscillatory phenomena such as standing or travelling waves are seen. Near the boundaries in parameter space between these two kinds of instability we expect some sort of interaction between steady and oscillating patterns. These interactions may well lead to complex dynamics. In this paper we consider such an interaction and derive, by symmetry arguments, the normal form for the behaviour close to the pattern-forming instability threshold when the system parameters are near the boundary which separates the regions of steady and oscillatory pattern-forming behaviour.

Much of the time this transition from steady instability to oscillatory instability takes place at a point where the preferred wavenumbers (for a spatially-extended plane layer) for the two kinds of instability are distinct; there is a jump in the preferred horizontal scale of the pattern. In this paper we examine the case where the ratio of the preferred wavenumbers is $1:\sqrt{3}$. Although this seems very restrictive, the normal form we derive is the simplest possible one for a Hopf/steady-state mode interaction on a hexagonal