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The bifurcation of heteroclinic cycles in systems of hydrodynamical type

Pascal Chossat[1]

[1]I.N.L.N. (CNRS) Sophia Antipolis
06560 Valbonne, FRANCE

Abstract. Robust heteroclinic cycles are a natural phenomenon arising in systems with symmetry and their bifurcation from a trivial state has been much studied since a seminal paper by Guckenheimer and Holmes (1988) [11]. Their existence is however subjected to a number of conditions often involving higher codimension. It turns out that these conditions are met in many hydrodynamical systems. I will explain this fact by the special form of the nonlinearities (advection terms) and illustrate it on systems with spherical symmetry.

Keywords. Heteroclinic cycles, hydrodynamics, symmetry, bifurcation, Navier-Stokes.
AMS (MOS) subject classification: This is optional. But please supply them whenever possible.

1 Introduction

It has been recognized about 12 years ago that certain intermittent-like behaviour of fluid flows could be related to the presence, in the dynamics of the model equations, of a stable *heteroclinic cycle*. This flow-invariant set is made of the union of heteroclinic orbits cyclically connecting a family of equilibria. In general, heteroclinic connections break under small perturbations. If however the system possesses certain geometric features such as symmetries, which imply the existence of flow-invariant subspaces for the amplitude equations, then it may happen that two equilibria lying in that subspace are connected by a saddle-sink connection, which is robust. If in addition a sequence of such orbits connects a set of equilibria in such a way that the set of connections closes back to itself, then one has obtained what is called a robust heteroclinic cycle. This situation was first analyzed in the context of population dynamics (see [12]).
Examples of robust heteroclinic cycles which bifurcate in hydrodynamical systems are well-known, and our aim here is not to describe new examples. An abundant literature has already been devoted to the study of these objects (see [8]). However there is one point which seemingly has not been made clear: despite the fact that, in addition to symmetry, a set of conditions (sometimes non-generic) have to be satisfied in order for robust heteroclinic cycles to bifurcate, it seems that such bifurcations frequently occur