Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **8 (2001)** 559-574

Computation of Normal Forms for Higher Dimensional Semi-Simple Systems

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Abstract. An efficient method to calculate the normal form and the associated nonlinear transformations for the semi-simple case is given in this paper. The one step transformation concept is adopted to make the approach very easy to be programmed. An intelligent judgement is used to simplify the tedious calculation. This method can be used to calculate high order normal form (without limitation, up to the capacity of the computer) of high dimensional (until dimension 9) ordinary differential equations of the nonlinear oscillators. A program in Mathematica language is designed to perform the calculation. Six examples are given in order to verify the method and to show the efficiency of the program.
Keywords. Normal form, Semi-simple bifurcation, Mathematica program, Symbolic computation, Nonlinear transformation
AMS (MOS) subject classification: Theory of dynamical system.

1 Introduction

The normal form method has been widely used in the fields of dynamical system, ordinary differential equations and nonlinear vibration. Duo Wang [6] made a very good summary from the viewpoint of mathematician and gave a detailed introduction to three basic methods of calculating normal form of ordinary differential equations. The three main methods are the matrix representation method; the adjoint operator method and the method based on representation theory of sl(2, R). Normal form theory plays an important role in the study of dynamical behavior of nonlinear systems near the dynamic equilibrium points because it greatly simplifies the analysis and formulations. This simple form can be used conveniently for analyzing the dynamical behavior of the original system in the vicinity of equilibrium. However, it is not a simple task to calculate the normal form for some given ordinary differential equations (ODE). In particular, it is difficult to derive the explicit formulas of a normal form in terms of the coefficients of the original nonlinear system. Therefore, the crucial part in computing normal forms is to find the coefficients of the normal forms and the associated nonlinear transformations efficiently. Furthermore, algebraic manipulations become very involved as the order of normal forms and the dimension of ODE increases. Thus, symbolic computations using symbolic computer languages such as Maple, Mathematica and Macsyma have been introduced to computing the normal