

## MIXED $H_2/H_\infty$ CONTROL OF MULTIRATE SYSTEMS

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**Abstract.** A mixed  $H_2/H_\infty$  problem is formulated and solved for multirate systems. The problem is motivated by its relation to robust  $H_2$  performance. In analogy with the single-rate problem, the solution is stated in terms of a parameterization of the optimal controller as a periodic observer-based controller, which reduces the optimal controller synthesis to parametric optimization of the estimator gains. The optimization problem can be characterized as a finite-dimensional convex optimization problem involving a set of matrix inequalities over the system period.

**Keywords.** Mixed  $H_2/H_\infty$  control; multirate systems; optimal control; periodic systems; robust control.

## 1 Introduction

Mixed  $H_2/H_\infty$  control problems have been proposed in order to design controllers which combine  $H_2$  performance objectives and  $H_\infty$ -type robustness criteria. Mixed  $H_2/H_\infty$  controllers have been designed for both continuous-time [4, 15, 9] and discrete-time systems [11, 13, 27], as well as sampled-data systems [28]. An important motivation for the mixed  $H_2/H_\infty$  problem stems from its relation to robust  $H_2$  performance [26, 31, 27]. The solution of the mixed  $H_2/H_\infty$  control problem can be stated in terms of a set of synthesis equations [4, 9, 27, 32], or by formulating the problem as a convex optimization problem [15, 13].

Multirate systems appear in digital control systems when different parts of the system operate with different rates. The rapid development of computer technology has greatly facilitated the implementation of such systems, due to which multirate control problems have received a considerable interest recently. The theory of multirate systems goes back to the work by Kranc [16], Kalman and Bertram [12] and Friedland [10]. More recent work on multirate systems is concerned with e.g. optimal control of multirate systems with a quadratic cost function [1, 3, 5, 7, 18, 19, 21, 29, 30], and  $H_\infty$  control of multirate systems [6, 23, 24, 29, 30].

The lifting technique is a standard procedure that is commonly applied to periodic multirate systems to represent the periodic system by a time-invariant, lifted system, e.g. [6, 20, 21, 29, 30]. The lifted system has as its