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LIFE SPAN OF SOLUTION FOR THE CAUCHY PROBLEM TO NONLINEAR REACTION-DIFFUSION SYSTEM

Xiaohua Xiong

Department of Computational Science,
Graduate School of Natural Science and Technology,
Kanazawa University, Kakuma-machi, Kanazawa 920-1192, Japan

Abstract. In this paper, we discuss life span of solution of Cauchy problem to nonlinear reaction-diffusion systems. By using Fourier transformation, fixed point theorem, energy and pointwise estimate methods, we obtain the existence of the solution. Estimates of the global solution and the life span of the local solution are also obtained.
Keywords. Nonlinear, reaction-diffusion system, life span, operator, fixed point theorem.

1 Introduction

We consider the Cauchy problem for the system of equations

$$\begin{cases} u_t - m_1\Delta u - m_2\Delta v = f_1(u, v, D_x u, D_x v, D_x^2 u, D_x^2 v) \\ v_t - m_3\Delta u - m_4\Delta v = f_2(u, v, D_x u, D_x v, D_x^2 u, D_x^2 v) \\ u(0, x) = \varphi(x), v(0, x) = \psi(x), \end{cases} \tag{1.1}$$

where $(t, x) \in \mathbf{R}_+ \times \mathbf{R}^n$, $\Delta = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}$, $D_x = (\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \cdots, \frac{\partial}{\partial x_n})$, $D_x^2 = (\frac{\partial^2}{\partial x_i \partial x_j}; i, j = 1, 2, \cdots, n)$ and $m_i (i = 1, 2, 3, 4)$ are constants. Assumptions to f_i $(i = 1, 2)$ and A_0 are the following:

*A*1) $f_i(\xi)$ $(\xi \in \mathbf{R}\times\mathbf{R}\times\mathbf{R}^n\times\mathbf{R}^n\times\mathbf{R}^{n^2}\times\mathbf{R}^{n^2})$ is smooth and $f_i(\xi) = O(|\xi|^{1+\alpha})$ $(|\xi| \to 0)$, where α is a positive integer and $i = 1, 2$.

*A*2) The matrix A_0 has the form $A_0 = \begin{pmatrix} a & c \\ 0 & b \end{pmatrix}$ or $A_0 = \begin{pmatrix} a & c \\ c & b \end{pmatrix}$ with the condition that $X^t A_0 X \geq \lambda_0 |X|^2$ for some integer $\lambda_0 > 0$ and any $X \in \mathbf{R}^2$.

In 1966, H.Fujita[1] considered the following scalar equation

$$u_t - \Delta u = u^{1+\beta}, \tag{1.2}$$