Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **9 (2002)** 139-151

THE MOTION OF A PARTICLE IN A FINITE INTERVAL SUBJECT TO AN INDEFINITE POTENTIAL AND THE STOCHASTIC COSNER CONJECTURE

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Abstract. Starting with the Langevin equation, the Ito calculus leads to the Fokker-Planck Equation,

$$\frac{\partial p}{\partial t} = \frac{\partial}{\partial x} \quad \frac{\partial}{\partial x}(ap) - bp \quad , \tag{1}$$

describing the motion of a particle undergoing diffusion $a(x,t)$ and drift $b(x,t)$. We assume that the particle is constrained to move in a finite interval of the real line $[s_1, s_2]$, having reflective or absorbing barriers at the end points. In either case, we provide conditions to show that if the particle is subjected to an indefinite potential $m(x)$, then there exists a probability density function,

$$p(x,t) = u(x)e^{-\lambda m(x)t}, \tag{2}$$

satisfying (1) with $\lambda \geqslant 0$. Some interesting cases arise when $m(x)$ is the inducing agent to the drift $b = 2\alpha a m_x$. We state the one dimensional stochastic Cosner conjecture and show its probabilistic implications on the dynamics of the Langevin particle. We reach our formulations through much more elegant means than was achieved before. In the process, we prove the conjecture for a large class of problems, and propose directions that should elucidate the general one dimensional case.

AMS classification: 49J20, 49J40, 49K20, 65N25.

Key words: Fokker-Planck Equations, Indefinite Potential, Principal Eigenvalue.

1 Introduction

Among all approaches used to describe the continuous motion of a particle on the real line, none is easier than that of Langevin. The Langevin model separates the forces applied to the particle into a random force, and a determinate one. The random component is the Brownian motion due to diffusion, while the determinate component is due to the drift (See [1] or [13]). Langevin obtained the first instance of what are called stochastic differential equations (SDE's):

$$dx = dx(t) = \sigma dw(t) + bdt. \tag{1.1}$$