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ON THE ASYMPTOTIC BEHAVIOUR OF DIFFERENCE EQUATIONS WITH CONTINUOUS ARGUMENTS

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Abstract. In this paper we study the asymptotic behaviour of solutions of difference equations with continuous argument, and obtain asymptotic estimates, which can imply asymptotic stability or stability of solutions.

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1. Introduction

Difference equations in continuous time appear quite frequently in various mathematical models [6]. Therefore, in the last few decades the qualitative theory of difference equations and inequalities in continuous time has attracted growing attention. Recently, the oscillation of solutions of this kind of equations have been extensively studied by many authors. For example, the reader is referred to [3], [4], [11], [12], [13]. In the papers [2], [8], [10] the existence of periodic solutions is investigated, whereas in the papers [1], [5], [14] stability conditions of solutions are given. The paper by Zhou and Yu [15] deals with the asymptotic properties of solutions of difference equation with continuous arguments.

Consider the difference equation with continuous arguments

$$x(t) = A(t)x(t-1) + B(t)x(p(t)) \tag{1}$$

where

(H_1) $A(t)$ is a real function such that $0 < A(t) < 1$;

(H_2) $B(t)$ is a real function;

(H_3) $p(t)$ is a real function such that $p(t) < t - \delta$ for the arbitrary small positive real number δ and $\lim_{t\to\infty} p(t) = \infty$.

The purpose of this paper is to study the asymptotic behaviour of solutions of difference equation (1), and apply our results to the particular cases, such as $p_1 t \le p(t) \le p_2 t$, for real numbers $0 < p_1 \le p_2 < 1$ and $\sqrt[p_2]{t} \le p(t) \le \sqrt[p_1]{t}$, for natural numbers $1 < p_1 \le p_2$. The new results obtained for difference equations with continuous arguments generalize results