POSITIVE SOLUTIONS OF A CLASS OF NEUTRAL DIFFERENCE EQUATIONS

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Abstract. The authors consider the first order neutral difference equation

$$\Delta \left(x_n - \frac{r_n}{r_{n-\tau}}x_{n-\tau} \right) + q_n x_{n-\sigma} = 0, \ n = 0, 1, 2, \cdots,$$

where $q_n \geq 0$, $\{r_n\}$ is nondecreasing, $\tau \geq 1$ and σ are integers, and $\sum_{n=1}^{\infty} \frac{1}{r_n} = \infty$. They prove that each positive solution of Eq.(1) can be classified as one of four types and give necessary and sufficient conditions for the existence of solutions of three of these types and a necessary condition for the existence of a solution of the fourth type. Examples of the results and suggestions for future research are included.

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1 Introduction

Consider the following first order neutral difference equation

$$\Delta \left(x_n - \frac{r_n}{r_{n-\tau}}x_{n-\tau} \right) + q_n x_{n-\sigma} = 0, \ n = 0, 1, 2, \cdots, \tag{1}$$

where the operator Δ is defined by

$$\Delta y_n = y_{n+1} - y_n,$$

and:

(i) $\{q_n\}$ is a sequence of nonnegative real numbers, $\{q_n\}$ is not eventually zero;

(ii) $\{r_n\}$ is a nondecreasing sequence of positive numbers;

(iii) $\tau \geq 1$ and σ are integers;

(iv) $\sum_{n=1}^{\infty} \frac{1}{r_n} = \infty$.