A BOUNDARY VALUE PROBLEM FOR FIRST ORDER IMPULSIVE INTEGRO-DIFFERENTIAL EQUATIONS IN BANACH SPACES*

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Abstract. This paper studies a boundary value problem for first order nonlinear impulsive integro-differential equations of mixed type on an infinite interval with an infinite number of impulsive times in a Banach space. By establishing a comparison result and utilizing the method of upper and lower solutions, criteria on the existence of maximal and minimal solutions of the boundary value problem are obtained.

AMS (MOS) subject classifications: 34A37, 34C25, 45J05

1 Introduction

Impulsive integro-differential equations arise naturally from a variety of applications, such as ecology, modelling of infectious disease, biological engineering and process control. Many of such equations may be studied in a general framework in abstract spaces. Consequently, the theory of impulsive integro-differential equations in abstract spaces has become a important area of research in recent years. For a detailed account on the basic theory and some of the recent development on impulsive integro-differential equations in abstract spaces, see monograph [4] and references therein. In a recent article [1], the periodic boundary value problem for first order nonlinear impulsive integro-differential equations of mixed type in a real Banach space E is studied. The problem is defined on a finite interval with a finite number of impulsive times. In this paper, we shall investigate a boundary value problem for such equations on an infinite interval with an infinite number of impulsive times in E. The paper is organized as follows. In Section 2, we prove several lemmas and establish a comparison result. By using the method of upper and lower solutions and the fixed point theorem, we then obtain, in Section 3, some criteria on the existence of maximal and minimal solutions of the