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# PERIODIC SOLUTIONS OF SECOND ORDER BOUNDARY VALUE PROBLEMS WITH IMPULSES [1]

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**Abstract.** This paper deals with periodic solutions of impulsive second order boundary value problems with parameter. Existence results are presented for nonlinearity with mixed sublinear and superlinear term at $\lambda = 0$ and (or) $\lambda \neq 0$. Continua of the solution set are obtained. The main technique is fixed point argument of expansion and compression type.
**Keywords.** impulsive, boundary value problems, periodic solutions, fixed point.
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## 1 Introduction and the Main Results

Let $M > 0$ be a constant, $J = [0,T]$, and $p \in C^1[0,T], p(t) > 0, t \in (0,T)$. Consider the following periodic boundary value problem with impulses at fixed moments

$$\begin{cases} -Lx + \frac{M^2}{p^2(t)} = f_\lambda(t, x(t)), & t \in J' = J\backslash\{t_1, t_2, \cdots, t_n\} \\ -\Delta(px')|_{t_k} = L_k(x(t_k)), & k = 1, 2, \cdots, n \\ \Delta x|_{t_k} = \hat{L}_k(x(t_k)), & k = 1, 2, \cdots, n \\ x(0) - x(T) = x_0, \quad -(px'|_0 - px'|_T) = x_1, & \end{cases} \tag{1.1}$$

where $(Lx)(t) = \frac{1}{p(t)}(p(t)x'(t))'$, $\lambda$ is a parameter and $f_\lambda(t,s) = f(\lambda, t, x)$. For impulsive periodic boundary value problems, Wei [7] introduces the notion of Green's function, and obtain maximal and minimal solutions in the case of $p(t) \equiv 1$ provided a pair of upper and lower solutions exist. In a recent paper, Cabada, A., Nieto, J.J., Franco, D. and Trofimchuk, S.I. in [1] study problem (1.1) in the case when $f$ is a Carathéodory function. By some technique based on the Banach contraction principle the authors develop a monotone iterative technique. They also get existence results if there exist upper and lower solutions.

In the present paper, we will study problem (1.1) in a different approach. We do not assume the pre-existence of upper and lower solutions. Instead,

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