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ON FIRST ORDER IMPULSIVE DIFFERENTIAL INCLUSIONS WITH PERIODIC BOUNDARY CONDITIONS

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Abstract. In this paper the concepts of lower and upper solutions combined with a fixed point theorem for condensing maps are used to investigate the existence of solutions for first order impulsive differential inclusions with periodic boundary conditions.

Keywords. Periodic boundary value problem, Impulsive differential inclusions, Convex multivalued map, Condensing map, Fixed point, truncation map, Upper and lower solutions.

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1 Introduction

This paper is concerned with the existence of solutions for the impulsive periodic multivalued problem:

$$y' \in F(t,y), \quad t \in J = [0,T], \ t \neq t_k, \quad k = 1,\dots,m, \tag{1}$$

$$\Delta y|_{t=t_k} = I_k(y(t_k^-)), \quad k = 1,\dots,m, \tag{2}$$

$$y(0) = y(T), \tag{3}$$

where $F : J \times \mathbb{R} \longrightarrow 2^{\mathbb{R}}$ is a compact and convex valued multivalued map, $0 = t_0 < t_1 < \ldots < t_m < t_{m+1} = T$, $I_k \in C(\mathbb{R},\mathbb{R})$ $(k = 1,2,\dots,m)$, $\Delta y|_{t=t_k} = y(t_k^+) - y(t_k^-)$, $y(t_k^-)$ and $y(t_k^+)$ represent the left and right limits of $y(t)$ at $t = t_k$, respectively.

The method of upper and lower solutions has been successfully applied to study the existence of solutions for impulsive initial and boundary value problems of first order. This method generates solutions of the problem, located in an order interval with the upper and lower solutions serving as bounds. Moreover, this method coupled with some monotonicity type hypotheses,