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REACHABILITY ANALYSIS for UNCERTAIN SYSTEMS - the ELLIPSOIDAL TECHNIQUE

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Abstract. The paper deals with reachability for linear systems subject to unknown but bounded disturbances, with hard bounds on the controls and the unknowns, as described earlier by the authors. An important concern is that the reach sets are achieved through closed-loop controls. The direct methods of finding the exact closed-loop reach sets look for them either as level sets for the solution of a "forward" Hamilton-Jacobi-Bellman-Isaacs equation or as set-valued solutions to an evolution equation of the integral funnel type. But direct schemes are difficult to realize. This justifies the introduction in this paper of ellipsoidal-valued approximations (both external and internal) for reach sets under uncertainty. The ellipsoidal techniques given here allow effective calculation of reach sets under uncertainty and other related items.

Keywords. reachability, reach sets, closed-loop control, HJBI equation, funnel equations, dynamic programming.

AMS (MOS) subject classification: 34H05, 34G25, 35F10, 49L, 49N70, 91A23.

1 Introduction

We discuss a key problem of control theory—the computation of domains reachable by a controlled process through available controls. The emphasis is on the fact that the system is not completely known. The system is subject to unknown but bounded disturbances and the information on its parameters may not be complete. The requirement is to describe the states reachable by the system *despite the disturbances* or incomplete information or, if exact reachability is impossible, to find the *guaranteed errors* for reachability [18],[10]. The theory of reachability under uncertainty is therefore more complicated than in its absence [13],[16],[20]. A crucial element here is that one has to distinguish reachability under closed-loop and open-loop controls. That is, for achieving effective results, the system has to be governed by *closed-loop controls*. This gives substantial improvement as compared to nonanticipative ("minmax") open-loop controls.

A detailed description of reachability under uncertainty for systems with linear structure and hard bounds on the unknowns is given in [10]. There it was indicated that the sets reachable under uncertain disturbances may be described either as level sets of the solution to a first order PDE of the "forward"