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Periodic Solutions for Nonlinear Evolution Equations

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Abstract. In this paper, we study the existence of periodic solutions of following type evolution equations

$$f(t, u(t)) \in -au''(t) + bu'(t) + A(t)u(t), \quad t \in R,$$

in infinite dimensional spaces. Our main results generalize the corresponding results in Browder [6], Hirano [16], Shioji [25] and Vrabie [26].

Keywords. Periodic solution, class (S_+), evolution equation, topological dgree.

AMS (MOS) subject classification: 1991 Mathematics Subject Classification: Primary 34C25, 34G20; Secondary 47H05, 47H10.

1 Introduction

The purpose of this paper is to study the existence of periodic solutions of the following type evolution equation

$$f(t, u(t)) \in -au''(t) + bu'(t) + A(t)u(t), \quad t \in R, \tag{$E1.1$}$$

in real reflexive Banach spaces or Hilbert spaces. First, in Section 2, we set $a = 0, b = 1$ and study the periodic solution of the first order evolution. Periodic solutions of the first order equations have been extensively studied by many authors. For the operator A in linear cases, see, Amann [1], Becker [5], Pruss [23], etc., and, for the operator A in nonlinear cases, see Browder [6], Chen [11], Deimling [12], Hirano [16], Shioji [25], Vrabie [26], etc.

In this paper, we consider that A is a multi-valued operator of the class (S_+) which was introduced by Petryshyn [22]. This condition is different from

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