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STABILITY RESULTS FOR THE ISHIKAWA ITERATION PROCEDURES

H. Y. Zhou[1], S. S. Chang[2], R. P. Agarwal[3] and Y. J. Cho[4]

[1]Department of Mathematics
Shijiazhuang Mechanical Engineering College
Shijiazhuang 050003, P. R. China
[2]Department of Mathematics
Sichuan University
Chengdu, Sichuan 610064, P. R. China
[3]Department of Mathematical Sciences
Florida Institute of Technology
Melbourne, FL 32901-6975, U.S.A.
[4]Department of Mathematics
Gyeongsang National University
Chinju 660-701, Korea

Abstract. Let X be a real uniformly smooth Banach space, K be a nonempty closed convex subset of X and $T : K \to K$ be a generalized Lipschitzian and hemicontractive mapping. It is shown that the Ishikawa iteration procedures are weakly $T-$stable. As consequences, several weak stability results are established.

Keywords. Hemicontraction, generalized Lipschitz condition, the Ishikawa iteration procedure, weakly $T-$stable.

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1 Introduction

Let X be a real Banach space with norm $\|\cdot\|$ and X^* be the dual space of X. The normalized duality mapping $J : X \to 2^{X^*}$ is defined by

$$J(x) = \{x^* \in X^* : \langle x, x^* \rangle = \|x\|^2 = \|x^*\|^2\},$$

where $\langle \cdot, \cdot \rangle$ denotes the generalized duality pairing. It is well known that, if X is uniformly smooth, then J is single-valued, $J(tx) = tJx$ for all $t > 0$ and $x \in X$, and J is uniformly continuous on bounded subsets of X ([1], [2]). We denote the single-valued normalized duality mapping by j.

An operator T with domain $D(T)$ and range $R(T)$ in X is said to be *generalized Lipschitzian* if there exists a constant $c > 0$ such that

$$\|Tx - Ty\| \leq c(1 + \|x - y\|) \tag{1.1}$$

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