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EXPONENTIAL STABILITY OF THE STEADY STATE OF REACTION DIFFUSION HOPFIELD NEURAL NETWORKS

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Abstract. In this paper, we study exponential stability of the steady state of a reaction diffusion Hopfield neural network by the method of upper and lower solutions. The study is done under the condition that the reaction term is quasi-monotone increasing. We further describe the domain of attraction and discuss certain points concerning the proof of the stability of the steady state for both quasi-monotonically decreasing and mixed reaction terms.

Keywords: Reaction diffusion, Hopfield neural networks, steady state, exponential stability.

1 Introduction

Recently, there has been considerable interest in the qualitative analysis of Hopfield neural networks [1, 2, 3, 4, 5, 6, 9, 10, 11, 15, 19, 22, 24], which can be described by nonlinear differential equations [6, 22]. Hopfield neural network model is very crucial for its various applications, such as associative memories and optimization. In [1, 2, 3, 4, 5, 9, 10, 11, 15, 19, 24], many sufficient conditions for the stability of hopfield neural network are given; however, this is considered over the change in time direction only. Strictly speaking, when electrons are moving in a non-uniform electromagnetic field, diffusion is inevitable. It is more common to study the neural network with reaction and diffusion, especially in chemistry as described in [16, 17, 18, 20]. This particular type of network is also studied in [1, 3, 7, 9, 12, 13, 14, 26, 27, 28]. In [9], the model of reaction diffusion hopfield neural network is given by

$$\begin{cases} C_i \dfrac{\partial u_i}{\partial t} = \sum\limits_{k=1}^{m} \dfrac{\partial}{\partial x_k}(D_{ik}(t,x,u)\dfrac{\partial u_i}{\partial x_k}) + \sum\limits_{i=1}^{n} T_{ij} g_j(u_j) - \dfrac{u_i}{R_i} + I_i \\ \dfrac{\partial u_i}{\partial n} = 0, \quad t \in I, \quad x \in \partial\Omega, \qquad i = 1, 2, \cdots, n, \end{cases} \tag{1}$$

where C_i is the capacitance, R_i is the resistance, I_i is the current, u_i is the voltage, g_j is the output function, D_i is the diffusion function, $T = (T_{ij})_{n\times n}$ the is weight matrix, $\Omega \in \mathbb{R}^m$ is a compact set, $mes\Omega > 0, I = [0, +\infty)$