Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **9 (2002)** 541-549

AN EXISTENCE THEOREM ON UNBOUNDED INTERVALS FOR A CLASS OF SECOND ORDER FUNCTIONAL DIFFERENTIAL INCLUSIONS IN BANACH SPACES

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Abstract. In this paper, we investigate the existence of solutions on infinite intervals to second order initial value problems for a class of functional differential inclusions in Banach spaces. We shall rely on a fixed point theorem due to Ma which is an extension to multivalued maps on locally convex topological spaces of Schaefer's theorem.
AMS (MOS) subject classifications: 34A60, 34G20, 34K40.

1 Introduction

Existence of solutions on compact intervals for functional differential equations has received much attention in recent years, we refer, for instance, to the books of Erbe, Qingai and Zhang [4], Hale [5] and Henderson [6], the papers of Hristova and Bainov [7], Nieto, Jiang and Jurang [12], Ntouyas [13] and Ntouyas and Tsamatos [14].

In the above mentioned papers the main tools used for the existence of solutions are the monotone iterative method combined with upper and lower solutions or the topological transversality theory of Granas. For more details on these theories we refer the interested reader to the the book of Ladde, Lakshmikantham and Vatsala [9] and to the monograph of Dugundji and Granas [3].

In this paper we shall give an existence result of solutions on infinite intervals to the initial value problem (IVP for short) of the second order functional differential inclusion

$$(\rho(t)y'(t))' \in F(t, y_t), \quad \text{a.e. } t \in J = [0, \infty), \tag{1.1}$$

$$y_0 = \phi, \ y'(0) = \eta \tag{1.2}$$

where $F : J \times C([-r, 0], E) \longrightarrow 2^E$ is a bounded, closed, convex valued multivalued map, $\rho \in C(J, \mathbb{R}_+)$, $\phi \in C([-r, 0], E)$, and E a real Banach space with the norm $|.|$.