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Qualitative Analysis and Exact Solutions to the Burgers-Korteweg-de Vries Equation

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Abstract. In this paper, using the qualitative theory of ordinary differential equations, we give a qualitative analysis to a two-dimensional plane autonomous system which is equivalent to the Burgers-Korteweg-de Vries equation; then applying the theory of commutative algebra and the first-integral method, we obtain exact solutions to the Burgers-Korteweg-de Vries equation. The results indicate that the solutions obtained in the previous literature are the particular cases of our results.

Keywords: solitary wave, exact solution, Burgers-KdV equation, equilibrium point, Poincare transformation, commutative algebra.

AMS (MOS) Subject Classification: 13P10, 34C05, 34C20, 35Q53.

1 Introduction

During the past three decades, the Burgers' equation, Korteweg-de Vries (KdV) equation and Burgers-Korteweg-de Vries equation (Burgers-KdV) have attracted a lot of attention from a rather diverse group of scientists such as physicists and mathematicians, because these three equations not only arise from realistic physical phenomena, but also can be widely applied to many physically significant fields such as plasma physics, fluid dynamics, crystal lattice theory, nonlinear circuit theory and astrophysics ([1,10,19,20,22,24,26,30, 33,34]).

Consider the Burgers-KdV equation

$$u_t + \alpha u u_x + \beta u_{xx} + s u_{xxx} = 0, \tag{1}$$

where α, β and s are real constants with $\alpha\beta s \neq 0$. Equ. (1) is applied as a nonlinear model of the propagation of waves on an elastic tube filled with a viscous fluid [19], the flow of liquids containing gas bubbles [34] and turbulence [10,22]. It can also be regarded as a combination of the Burgers' equation and KdV equation, since the choices $\alpha \neq 0$, $\beta \neq 0$, and $s = 0$ lead equ. (1) to the Burgers' equation

$$u_t + \alpha u u_x + \beta u_{xx} = 0, \tag{2}$$

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