Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **9 (2002)** 555-573

NUMERICAL PROCEDURES FOR THE DETERMINATION OF AN UNKNOWN COEFFICIENT IN PARABOLIC DIFFERENTIAL EQUATIONS

Hossein Azari[1]

[1]Institute for Studies in Theoretical Physics and Mathematics (IPM)
P. O. Box 19395–5746, Tehran, Iran

Abstract. We consider several finite difference approximation to an inverse problem of determining an unknown source parameter $p(t)$ which is a coefficient of the solution u in a linear parabolic equation subject to additional information on the solution integral type along with the usual initial boundary conditions. The backward Euler scheme is studied and its convergence is proved via an application of the discrete maximum principle for a transformed problem. Error estimates for u and p involve numerical differentiation of the approximation to the transformed problem. Some experimental numerical results using the newly proposed numerical procedure are discussed
Keywords. Inverse problem, convergence, maximum principle, numerical method.
AMS (MOS) subject classification: Primary 35R30; Secondary 65M32, 76R50, 65M06.

1 Introduction

In this paper we study a finite difference method for approximating the unknown source parameter $p = p(t)$ and $u = u(x, t)$ of the following inverse problem. Find $u = u(x, t)$ and $p = p(t)$ which satisfy

$$\begin{aligned} u_t &= u_{xx} + p(t)u + f(x,t) \qquad \text{in} \quad Q_T, \\ u(x,0) &= \phi(x), \qquad 0 \le x \le 1, \\ u(0,t) &= g_1(t), \qquad\qquad 0 \le t \le T, \\ u(1,t) &= g_2(t), \qquad\qquad 0 \le t \le T. \end{aligned} \tag{1.1}$$

subject to the additional specification

$$\int_0^1 K(x)u(x,t)dx = E(t), \qquad 0 \le t \le T. \tag{1.2}$$

where $Q_T = \{(x,t) : 0 < x < 1, 0 < t < T\}$, $T > 0$, f, g_l, g_2, $E \neq 0$ and $K(x)$ are known functions. In addition, it is assumed that for some constant $\rho > 0$ the kernel $K(x)$ satisfies

$$\int_0^1 |K(x)|dx \le \rho. \tag{1.3}$$