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MODELLING AND CONTROL OF THE MOTION OF A DISK ROLLING ON A RIGID TERRAIN

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Abstract. This work deals with the modelling and control of the motion of a disk rolling on a given rigid terrain. It is assumed here that the motion of the disk is controlled by a tilting moment, a directional moment and a pedalling moment. First, a mathematical model, of the motion of the disk rolling on the terrain, is derived. Then, by using a kind of an inverse control transformation a control strategy is proposed under which the motion of the disk is stabilized and is able asymptotically to track any smooth trajectory which is located on the given terrain.

Keywords. Rolling disk, rigid terrain, nonholonomic constraints, asymptotic stability, asymptotic tracking.

AMS (MOS) subject classification: 93B 70E 70Q 93C

1 Introduction

This work deals with the modelling and control of the motion of a disk rolling on a given rigid terrain. It is assumed here that the motion of the disk is controlled by a tilting moment, a directional moment and a pedalling moment. Thus, to some extent this work is a continuation of [1], where such a problem is dealt with but where the motion takes place on a horizontal plane. For the control and guidance of the motion of a disk rolling on a horizontal plane see for example [2,3] and the references cited there.

The problem dealt with here is a kind of introduction to the modelling and control of the motion of a riderless bicycle, or any other wheeled vehicle, on a rigid terrain.

In this work, first, a dynamical model for the rolling disk is derived. Then, by using a kind of inverse control transformation, a procedure is proposed for the design of feedback control laws for the pedalling moment, the tilting moment and the directional moment, such that the disk's motion will be stabilized (that is, the disk's plane will be vertical to the tangent plane of the terrain, at the point of contact between the disk and the terrain), while simultaneously controlling its speed and direction in such a manner that the disk will be able to asymptotically track any smooth trajectory which is located on the given terrain.