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ANALYTICAL SOLUTION FOR SOLUTE TRANSPORT WITH FREUNDLICH SORPTION

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Abstract. Analytical solutions are derived for the solute transport model with Freundlich sorption under nonequilibrium conditions, without making any simplifying apriori assumptions. The solutions are in the form of traveling wave fronts and it is shown that in general, there exists four distinct wave solutions corresponding to four different wave speeds. The methodology of finding such solutions is outlined and an illustrative example is presented.
Keywords: Contaminant transport, Freundlich sorption, Nonequilibrium conditions, concentration profile, Analytical solution.

1 Introduction

The sorption process is known to play an important role in the transport and disposition of solutes in subsurface systems. There is an excellent review article by Weber *et al.* (1991) which explores the current level of understanding of sorption processes and the use of different models to describe them. Also, there are several other studies (Abriola, 1986; Cameron and Klute, 1986; Cederberg *et al.*, 1985; Hinz *et al.*, 1994; Notodarmojo *et al.*, 1991; van Genuchten and Wierenga, 1976; van Genuchten, 1981; van der Zee, 1990; Walsh *et al.*, 1984) involving various sorption isotherms to one dimensional transport models. However, most of these studies are either computational or based on simplifying assumptions such as linear sorption or local equilibrium so that analytical formulae for the concentration profiles of the solutes can be constructed. An exception is the work in van der Zee (1990), where some exact solutions were obtained for transport with either Langmuir or Freundlich isotherm under nonequilibrium conditions. But again, a restrictive assumption, such as neglecting the coupled (multiplicative) effect of dispersion and nonequilibrium was made on the mathematical model to obtain those exact solutions. Since, in the model equation, expressed with respect to the traveling wave coordinate, the term representing the coupled multiplicative effect appears in a product form with the second derivative of the aqueous phase concentration, even if the coupled multiplicative effect is

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