Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **10 (2003)** 863-874

ON THE ZEROS OF TRANSCENDENTAL FUNCTIONS WITH APPLICATIONS TO STABILITY OF DELAY DIFFERENTIAL EQUATIONS WITH TWO DELAYS

Shigui Ruan[†,1] and Junjie Wei[‡,2]

[†]Department of Mathematics
University of Miami
Coral Gables, FL 33124-4250, USA

[‡]Department of Mathematics
Harbin Institute of Technology
Harbin, Heilongjiang 150001, China

Abstract. In this paper, we first establish a basic theorem on the zeros of general transcendental functions. Based on the basic theorem, we develop a decomposition technique to investigate the stability of some exponential polynomials, that is, to find conditions under which all zeros of the exponential polynomials have negative real parts. The technique combines the D-decomposition and τ-decomposition methods so that it can be used to study differential equations with multiple delays. As an application, we study the stability and bifurcation of a scalar equation with two delays modeling compound optical resonators.

Keywords. Transcendental polynomials, delay differential equations, stability, bifurcation, compound optical resonators.

AMS (MOS) subject classification: 30C15, 34K20.

1 Introduction

For an ordinary differential equation, the trivial solution is asymptotically stable if and only if all roots of the corresponding characteristic equation of the linearized equation have negative real parts. Since the characteristic function is a polynomial, the well-known Routh-Hurwitz criterion can be used to determine the negativity of the real parts of the characteristic roots. Similar equivalence holds for delay differential equations, especially the discrete delay differential equations. However, the characteristic functions corresponding to

[1]Research was partially supported by the NSERC of Canada and a start-up fund from the College of Arts and Sciences at the University of Miami. On leave from Dalhousie University, Halifax, Nova Scotia, Canada.

[2]Research supported by the National Natural Science Foundations of (NNSF) China.