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ON THE RECURSIVE SEQUENCE $x_{n+1} = \alpha_n + \frac{x_{n-1}}{x_n}$ II

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Abstract. In this paper we investigate the boundedness character and the periodic nature of the positive solutions of the difference equation

$$x_{n+1} = \alpha_n + \frac{x_{n-1}}{x_n}, \quad n = 0, 1, 2, ...,$$

where α_n is a two periodic sequence of nonnegative real numbers and where the initial conditions x_{-1} and x_0 are arbitrary positive real numbers.
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1 Introduction and some basic observations

In [3] Ladas offered the following conjecture:

Conjecture A. *Every positive solution of*

$$x_{n+1} = \alpha + \frac{x_{n-1}}{x_n}, \quad n = 0, 1, 2, ... \tag{1}$$

where $\alpha \geq 0$ *is bounded if and only if* $\alpha \geq 1$. *Furthermore if* $\alpha = 1$ *every positive solution of Eq.*(1) *converges to a two-cycle and if* $\alpha > 1$ *every positive solution of Eq.*(1) *converges to* $\alpha + 1$.

Soon after the conjecture was confirmed in [1]. The following two open questions were posed by professor Ladas in a private communication:

Question 1. Consider the difference equation

$$x_{n+1} = \alpha_n + \frac{x_{n-1}}{x_n}, \quad n = 0, 1, 2, ... \tag{2}$$

where α_n is a sequence of nonnegative real numbers which converges to $\alpha \geq 0$. What do the solutions with positive initial conditions x_{-1} and x_0 do?

Question 2. Assume that the sequence (α_n) in Eq.(2) is periodic with prime period two and nonnegative. What do the solutions with positive initial conditions x_{-1} and x_0 do?