EXTREMAL SOLUTIONS FOR FIRST ORDER SINGULAR PROBLEMS WITH DISCONTINUOUS NONLINEARITIES

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Abstract. We derive sufficient conditions for the existence of extremal solutions for first order ordinary differential equations whose right–hand side may be discontinuous and singular. Furthermore, the initial conditions that we consider may be located exactly over one of the singularities.
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1   Introduction

In this paper we consider the non-local Cauchy problem

$$u'(t) = f(t, u(t)) \text{ for a.e. } t \in [a, b], \quad u(a) = u_a, \tag{1.1}$$

where $u_a \geq 0$. We are interested in nonlinearities f which can be discontinuous in both variables t and u and singular at $u = 0$ and $t = a$. Using the lower and upper solutions method we prove, under appropriate assumptions, existence of extremal solutions of problem (1.1).

Among the first works devoted to lower and upper solutions for an initial value problem, we can quote the papers of G. Peano [14] in 1885 and O. Perron [15] in 1915. They consider the problem

$$u'(t) = f(t, u(t)) \quad \text{for all } t \in [a, b], \quad u(a) = u_a,$$

with f continuous, and prove the existence of extremal solutions between two continuous functions $\alpha \leq \beta$ which today we would call lower and upper

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