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STABILITY OF A GENERAL LINEAR DELAY-DIFFERENTIAL EQUATION WITH IMPULSES

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Abstract. In this paper we establish some sufficient conditions and also a necessary condition for asymptotic stability in a general linear delay-differential equation with impulses.
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1 Introduction

The study of certain ordinary differential equations with impulses was initiated in the $1960's$ by a seminal paper by Milman and Myshkis (see [29], [30]). After a period of active research, mostly in Eastern Europe from 1960-1970, culminating with the monograph by Halanay and Wexler [21], the subject received little attention during the seventies. An important monograph was presented by Pandit and Deo [31] in 1982, and two books by Bainov and Simeonov [9], [10], who present the state of the art in the theory of such systems. However, comparatively, not much has been done in the study of functional differential equations with impulses. In recent years, many examples of differential equations with impulses have arisen in several areas of applications and contexts. In the periodic treatment of some diseases, impulses correspond to administration of a drug treatment or a missing product. In environmental sciences, seasonal changes of the water level of artificial reservoirs, as well as under the effect of floodings, can be modeled as impulses. See for example [32], [33], [35], and more specifically [18]. We study the asymptotic stability of the zero solution of our original equation, and show that under some simple conditions all solutions of the equation with impulsive effect will be asymptotically periodic. The results are transferable to the case of stationary linear compartment systems with pipes, see [19], [20]. Most of the efforts seem to have been devoted to understanding the initial value problem associated with such a system (see, e.g. Anokhin [3], Anokhin and Braverman [4]). Nonetheless, in Gopalsamy and Zhang [17], preliminary stability and oscillation results are presented for the case of a single delay under the strong condition that the delay is smaller than the length of the