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STOCHASTIC LQ CONTROL WITH GENERALIZED COVARIANCE CONSTRAINTS

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Abstract. We consider the stochastic LQ control subject to the generalized covariance constraints (GCC) problem for both the discrete-time and the continuous-time Linear Time-invariant systems. It is shown that the feasibility of the GCC problem is equivalent to the feasibility of certain linear matrix inequalities (LMI). If the LMIs are feasible, the feasible controller set can be parameterized by the solutions to the LMIs. In addition, LQ performance can be optimized over the controller set and the global optimal solution can be found by semidefinite programming.
Keywords. Generalized covariance control; Linear Matrix Inequality; LQ control; LQG; Semi-definite programming;

1 Introduction

Variance control plays an important role in stochastic control theory and a number of controller design methods have been proposed and are well-known. Minimization of variances of certain process variables [1] is well justified in process control because the reduction of the variances of quality variables not only means improved product quality but also makes it possible to increase throughput, reduce energy consumption and save raw materials. However, minimizing the variances of some process variables alone may result in unacceptable control input variance [10]. Minimum variance control can be treated as a singular case of Linear Quadratic Gaussian (LQG) controller design [4] [9]. The latter is to minimize the scalar sum of the variances of the output and the input variables based on a known stochastic disturbance model. In LQG controller design, the weighting matrices are usually considered to be free design parameters, and proper choice of the weighting matrices is important for the closed-loop system to have satisfactory performance. Even though some rules of thumb to choose the weighting matrices have been given in [2], there is no systematic method of selecting the weighting matrices to ensure good overall closed-loop performance.

It is convenient and often necessary to set the variances or covariance constraints on the input and output variables directly in industrial applications. Traditional LQG controller design method can not achieve this straightforward goal. Several modified LQG controller design methods [12] [15] have been proposed to solve the variance constraints problem. The main idea of the modified LQG controller design is to iterate the weighting matrices until the variance constraints are satisfied. Mäkilä et al. [12] presented an iterative algorithm to minimize the LQ performance index subject to constraints on the variances of some variables specified by the designer, but the global convergence property of the algorithm is not known. Some self-tuning controller design methods [11] [18] were also proposed to explicitly restrict the variances of the state and the input variables based on adaptively adjusting the Lagrange multipliers of the variance constrained control problem.