Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **11 (2004)** 1-17
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AN (H, K) DICHOTOMY SPECTRUM FOR SYSTEMS OF LINEAR DIFFERENTIAL EQUATIONS WITH IMPULSE EFFECT

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Abstract. In this paper we introduce the notion of (h, k) dichotomy spectrum for systems of nonautonomous linear differential equations with impulse effect. This new kind of spectrum is based on the notion of (h, k) dichotomies and consists of a union of disjoint closed intervals. The spectral intervals and associated spectral manifolds generalize the concept of eigenvalues and eigenspaces of an autonomous differential equation $\dot{x} = Ax$ without impulse. The main result of this paper is a spectral theorem which describes all possible forms of the (h, k) dichotomy spectrum. Examples of spectra in the scalar case as well as some inclusion properties are given.

Keywords. (h, k) dichotomy, spectrum, impulse

AMS (MOS) subject classification 2000: Primary 34A37; Secondary 34A30, 34D09

1. Introduction

The notion of dichotomy spectrum for linear nonautonomous ordinary differential equations has been the subject of [17]. We generalize the results of [17] in two ways; we use the more general concept of (h, k) dichotomies and we consider a broader class of linear differential equations, namely linear differential equations with impulse effect, described by

$$\begin{aligned} \dot{x} &= A(t) \cdot x, & t \neq t_k,\ k \in \mathbb{Z}, \\ \Delta x(t_k) &= \tilde{A}(t_k) \cdot x(t_k), & t = t_k, \end{aligned} \tag{1.1}$$

where $\Delta x(t_k) = x(t_k^+) - x(t_k)$, represents a jump of the solution $x(t)$ at the instant $t = t_k$. Otherwise stated, a solution of the system (1.1) is a left-continuous function with countably many jumps, which happen at the times t_k, $k \in \mathbb{Z}$ and satisfy equation (1.1) identically. Thus, $\Delta x(t_k) = \tilde{A}(t_k) \cdot x(t_k)$ means that the solution jumps from $x(t_k)$ to $x(t_k^+) = (I + \tilde{A}(t_k)) \cdot x(t_k)$ at the instant t_k. We shall assume throughout this work that $A(t)$ is a locally

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[1]Support from Dirección de Investigación and Centro de Modelación y Computación Científica de la Universidad de La Frontera is acknowledged. Research Grant FONDECYT 1020938

[2]This paper was written while the second author was a member of the "Graduiertenkolleg: Nichtlineare Probleme in Analysis, Geometrie und Physik" (GRK 283) financed by the Deutsche Forschungsgemeinschaft and the State of Bavaria (Germany).