Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **11 (2004)** 69-80
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Existence of Positive Solutions and Oscillation for Difference Equations with Unbounded Delay

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Abstract. This paper considers the difference equation with unbounded delay

$$x_{n+1} - x_n + p_n x_{\tau(n)} = 0, \quad n = 0, 1, 2, \cdots ,$$

where $\tau : \; N \to Z$, $\tau(n) < n$ for $n \in N$ and $lim_{n\to\infty}\tau(n) = \infty$. Some new criteria of oscillation and existence of positive solutions for this equation are derived.

Keywords. Difference equations, unbounded delay, oscillation, positive solution.
AMS subject classification: 39A10

1 Introduction

Recently, many papers have devoted to the development of qualitative theory of difference equations [1-9]. Their significance is illustrated in applications involving random walk problems, mathematical physics problems, and numerical difference approximation problems, etc. To further the qualitative theory of difference equations, in this paper, we shall consider the difference equation of the form

$$x_{n+1} - x_n + p_n x_{\tau(n)} = 0, \quad n = 0, 1, 2, \cdots , \tag{1.1}$$

where $\{p_n\}$ is a real sequence, $\tau(n) < n$ for any $n \in N = \{0, 1, 2, \cdots\}$.
For the sake of convenience, let

$$a_n = \left(\frac{n-\tau(n)}{n-\tau(n)+1}\right)^{n-\tau(n)+1}, \quad n = 0, 1, 2, \cdots ,$$

$$\tau^0(n) = n, \; \tau^m(n) = \tau(\tau^{m-1}(n)), \; m = 1, 2, \cdots , \tag{1.2}$$

$$\tau^{-1}(n) = min\{m|\tau(m) \ge n, \; \tau(m+1) \ge n, \; \tau(m+2) \ge n, \cdots\}$$

and

$$\tau^{-(m+1)}(n) = \tau^{-1}(\tau^{-m}(n)), \; m = 1, 2, \cdots , \; n = 0, 1, 2, \cdots .$$