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STEADY STATES FOR NONLINEAR FOOD CHAIN MODEL WITH DEGENERATE SELF-POPULATION DIFFUSIONS

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Abstract. In this paper, we discuss the existence of positive solutions to a certain nonlinear food chain model with degenerate self-population diffusions under homogeneous Robin-Dirichlet boundary conditions. First we investigate the uniqueness of positive solutions to predator-prey interacting system between two species with degenerate self-diffusion rates under Robin-Dirichlet boundary conditions. Then we give sufficient and necessary conditions for the existence of positive solutions for a food chain model with degenerate self-diffusion rates. The method employed is a fixed point theory in a positive cone.
Keywords. Positive Solutions, Food-chain Model, Nonlinear Elliptic System, Degenerate Self-population Diffusion, Fixed Point Index.
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1 Introduction

Of concern is the existence of positive solutions of the food-chain model with degenerate self-population diffusions;

$$\begin{cases} u_t = \varphi_1(x,u)\Delta u + u(f(x,u) - a(x)v^2 - b(x)w^2) \\ v_t = \varphi_2(x,v)\Delta v + v(a(x)u^2 + g(x,v) - c(x)w^2) \\ w_t = \varphi_3(x,w)\Delta w + w(b(x)u^2 + c(x)v^2 + h(x,w)) \ \text{ in } \Omega \times (0,\infty), \\ B_1 u = B_2 v = B_3 w = 0 \ \text{ on } \partial\Omega \times (0,\infty) \\ u(x,0) = \tilde{u}_0(x) \geq 0, \ \ v(x,0) = \tilde{v}_0(x) \geq 0, \ \ w(x,0) = \tilde{w}_0(x) \geq 0 \ \text{ in } \Omega, \end{cases} \tag{1.1}$$

in a bounded region $\Omega \in \mathbf{R}^n$ with smooth boundary where $B_i u = \kappa_i \partial u/\partial n + \tau_i u$ are Robin-Dirichlet boundary conditions with $\kappa_i \geq 0$, $\tau_i > 0$ for $i = 1, 2, 3$. Δ is the Laplacian operator in $\mathbf{R}^n$, u, v and w represent the densities of distinct three species.

$a(x), b(x), c(x) \in C^1(\Omega)$ are nonnegative functions and some conditions on the birth rates f, g, h will be imposed later. (See H4.1 - H4.4.) The growth rates in the model (1.1) were motivated by the one in [8] and modified so that our system describes food chain interactions among three species. The functions φ_i, $i = 1, 2, 3$ play as degenerate self-population diffusions, namely, those could be vanished at some points in the domain Ω and depends on the