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## STABILITY OF THE STEADY STATE OF THE SECOND-ORDER HOPFIELD NEURAL NETWORKS WITH REACTION-DIFFUSION TERMS

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**Abstract.** Stability of the steady state solutions of the second-order Hopfield neural networks with reaction-diffusion terms are studied in this paper. Several fundamental issues are discussed by means of Lyapunov functions and the method of upper and lower solutions. Criteria on local asymptotic stability, global asymptotic stability, local exponential stability and global exponential stability are established.

**Keywords.** Second-order Hopfield neural networks, reaction-diffusion, steady state solution; stability.

## 1 Introduction

Since higher-order neural networks have better functions than first-order neural networks in terms of the approximation ability, the convergence rate, the storage capacity and the permissible mistake ability, the study of higher-order neural networks attracts a lot of attentions of researchers. Considerable effort has been put into the investigation of their analysis and synthesis in the past decade [1-5].

It is well known that engineering applications of Hopfield neural networks, such as optimization and association, rely crucially on the dynamical behaviors of the networks. Therefore, qualitative analysis of neurodynamics, such as fundamental properties of stability and oscillation, is indispensable for practical design of neural-network models and tools [6-11]. In neural association memories, for instance, the local asymptotically stable equilibrium states ( as attractors ) store information and constitute distributed and parallel neural memory networks. In such cases, the purpose of qualitative analysis is to study the state convergence so as to ensure the recall capability of the memories, and existence, number of states, asymptotical stability, and attracting regions of the equilibrium points for the storing capability of the networks [12-14]. On the other hand, to solve problems such as optimization, neural control, and signal processing, dynamical neural networks have

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