DETERMINATION OF AN UNKNOWN COEFFICIENT IN A PARABOLIC INVERSE PROBLEM

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Abstract. In this paper we study a parabolic inverse problem of finding $a(t)$ such that $u_t = a(t)u_{xx}$ subject to initial–boundary value conditions and an over–specified condition. We use the over–specified information to solve for the unknown function and then transform these inverse problems into some nonclassical equations in which a trace type of functional is involved. By applying the maximum principle, we deduce some a priori estimates for the solution of our nonclassical problem. Then the continuity method can be applied to establish the global existences of solutions to these problems.

Keywords. Inverse problem, convergence, maximum principle.

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1 Introduction

Let $T > 0$ and $Q_T = (0,1) \times (0,T]$. Consider the parabolic inverse problem of finding $a(t)$ and $u(x,t)$ such that the pair (a,u) statisfies

$$
\begin{aligned}
u_t &= a(t)u_{xx}, && \text{in } Q_T, \\
u(x,0) &= \phi(x), && x \in [0,1], \\
u(0,t) &= g_1(t), && t \in [0,T], \\
u(1,t) &= g_2(t), && t \in [0,T],
\end{aligned}
\tag{1.1}
$$