Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **12 (2005)** 1-22

PLANAR PLANFORMS WITH AND WITHOUT MIDPLANE REFLECTION

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Abstract. Systems of differential equations commuting with the action of the planar Euclidean group, with and without the midplane reflection symmetry $\mathbf{Z}_2^e$, are studied. Given a fixed planar lattice $\mathcal{L}$ with symmetry group $\Gamma = H \ltimes \mathbf{T}^2$, where H is the holohedry of $\mathcal{L}$ and $\mathbf{T}^2$ is the torus group of translations modulo $\mathcal{L}$, we compute the lattices of isotropy subgroups for large irreducible representations of Γ and $\Gamma \times \mathbf{Z}_2^e$. Until now, only the maximal isotropy subgroups of Γ and the lattice of isotropy subgroups for the smallest representations of Γ have been computed. The *axial* subgroups (i.e. the isotropy subgroups with one-dimensional fixed-point subspaces) are generically associated to *planforms* (i.e. steady state solutions) of the original differential equations by the Equivariant Branching Lemma. The computations of the axial subgroups of $\Gamma \times \mathbf{Z}_2^e$ give a proof of the existence of the three new planforms obtained in Dionne, Silber and Skeldon (Stability results for steady, spatially periodic planforms, *Nonlinearity* **10** (1997), 321–353).
Keywords. Bifurcation Theory, Symmetry, Planforms, Dynamical Systems.
AMS (MOS) subject classification: 37G40 Symmetries, equivariant bifurcation theory

1 Introduction

1.1 Previous Related Work

In [4], systems of differential equations commuting with the action of the planar Euclidean group are studied and *planforms* (steady-state spatially periodic solutions) bifurcating from a trivial equilibrium are found using the Equivariant Branching Lemma. The procedure used in [4] is the following. For each planar lattice $\mathcal{L}$, let $\Gamma = H \ltimes \mathbf{T}^2$, where H is the holohedries of $\mathcal{L}$ and $\mathbf{T}^2$ is the torus group of planar translations modulo $\mathcal{L}$. For each irreducible representation of Γ, the *axial* subgroups of Γ (i.e. isotropy subgroup with a one dimensional fixed point space) are computed. The Equivariant Branching Lemma asserts that generically there exists a branch of spatially periodic solutions bifurcating from a trivial equilibrium corresponding to each axial subgroup of Γ.

Note that Γ acts on the space of functions that are periodic with respect to the lattice $\mathcal{L}$ and that each irreducible subspace of this action is absolutely