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COEXISTENCE STATES OF CERTAIN POPULATION MODELS WITH NONLINEAR DIFFUSIONS AMONG MULTI-SPECIES

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Abstract. We discuss the coexistence states of certain population models with nonlinear diffusions among multi-species under homogeneous Dirichlet boundary conditions by using the method of *a system of upper-lower solutions*. We give sufficient conditions for the coexistence states and apply our results to three-species interacting systems with degenerate self diffusions or strictly positive cross diffusion rates.

Keywords. Coexistence states, degenerate self-diffusions, nonlinear diffusions, upper-lower solutions.

AMS (MOS) subject classification: 35J60, 35Q80

1 Introduction

In this paper, we are concerned with the coexistence states of componentwise nonnegative solutions to the general elliptic interacting systems with nonlinear diffusions: for $i = 1, ..., d$,

$$\begin{cases} -\Delta[\varphi_i(u_1, \cdots, u_d)u_i] = u_i f_i(x, u_1, \cdots, u_d) & \text{in } \Omega, \\ u_i = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where Ω is a bounded region in R^n with smooth boundary $\partial\Omega$. This is steady states of the general $d \times d$ biological interacting system with u_i representing the densities of d different species. The functions φ_i are the density-dependent diffusion rates and f_i the relative growth rates of the population u_i.

We say that two species are in *predator-prey* interaction if one of the growth rates involved is increasing in the prey while the other decreasing in the predator. Also two species are in *cooperation* if each of their relative growth functions is increasing in the other, and they are in *competition* if these functions are decreasing in the other one. Refer [14] for more details.

There has been a considerable amount of interest to a type of the system (1) since the proposal of the model in study of spatial segregation of two interacting species by Shigesada *et al.* [19]. In [11], the authors investigated the existence of non-constant solutions to the 2×2 competing interaction system of a type (1) when the diffusion and the growth rates are linear with respect to