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A Closed-Form Solution to a Dynamic Portfolio Optimization Problem

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Abstract. In this paper we consider a continuous-time Markowitz mean-variance type portfolio optimization problem where the variance is replaced by a Earnings-at-Risk (EaR) of terminal wealth. In a Black-Scholes setting of financial markets, we obtain closed-form expressions for best constant-rebalanced portfolio investment strategies and the mean-EaR efficient frontier.

Keywords. Dynamic portfolio selection; Earnings-at-Risk; constant-rebalanced portfolios; Black-Scholes model.

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1 Introduction

The pioneering work of Markowitz [9] introduced the mean-variance framework for portfolio selection and risk management which are important problems in investment finance. The mean-variance approach has become the foundation of modern finance theory and has inspired a significant number of extensions and applications. From a theoretical point of view, there are two challenges. The first is the generalization of the Markowitz mean-variance model to multi-period. A large part of literature focuses on maximizing some time-additive utility of terminal wealth and/or consumption; see, for example, Smith [13], Merton [10, 11] and Samuelson [12]. However, solving the dynamic mean-variance portfolio remains a challenging problem; see, e.g., Chen, Jen and Zionts [3]. Consequently, Markowitz's mean-variance formulation has not been fully explored in dynamic cases for a long time. It was only until recently where the dynamic mean-variance problems have been solved analytically; see Li and Ng [8] for the discrete-time case and Zhou and Li [14] and Emmer, Klüppelberg and Korn [5] for the continuous-time case.

The second challenge lies in the appropriateness of the risk measure. In the classical literature, variance is typically the relevant measure of risk. However, some deficiencies of using variance in quantifying risk have been documented. Markowitz also noted that there exists other possible choices