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DOUBLE HOPF BIFURCATIONS IN THE QUASIGEOSTROPHIC POTENTIAL VORTICITY EQUATIONS

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Abstract. We analyze the bifurcations that occur in a simple baroclinic flow described by the two layer quasigeostrophic potential vorticity equations. In particular, we study the solutions that bifurcate from a steady solution (basic state) that can become unstable due to both baroclinic and barotropic sources. It is found that, *via* Hopf bifurcation, travelling waves of various wave numbers can equilibrate when the steady solution becomes unstable. Furthermore, there are points in the space of control parameters where double Hopf bifurcations occur. Near these points, the nonlinear interaction of two travelling wave solutions is analyzed.

At the bifurcation points, centre manifold reduction and normal form theory are used to deduce the local behaviour of the full system of partial differential equations from a low-dimensional system of ordinary differential equations. Although it is necessary to numerically approximate the relevant eigenvalues and eigenfunctions, the projection onto the centre manifold and reduction to normal form can be done analytically. Thus, a combination of analytical and numerical methods are used to obtain numerical approximations of the normal form coefficients, from which the dynamics are deduced.

The analysis of the double Hopf bifurcations show that, in certain regions in the space of parameters, there is bi-stability of wave solutions, and hysteresis of the waves is observed. The quasi-periodicity that is possible in this type of bifurcation is not observed.

Keywords. baroclinic and barotropic instabilities, steady flow to wave flow transition, hysteresis of travelling waves, centre manifold reduction, numerical approximation of normal form coefficients.

AMS (MOS) subject classification: 37N10, 76U05, 86A10.

1 Introduction

It has long been argued that baroclinic instability is fundamental for the generation of mid-latitude waves in large-scale geophysical fluids [3, 6]. This instability process depends on the existence of vertical shear within a basic state (steady solution) of the fluid, which, in this context, implies the existence of a horizontal temperature gradient.

The (baroclinic) quasigeostrophic potential vorticity equations form a relatively simple model of mid-latitude flow that is often used to study this important source of instability (see *e.g.* [20]). The model is derived under the assumption that the characteristic length and velocity of the flows of interest