ERGODIC THEOREMS FOR ALMOST EXPANSIVE CURVES IN HILBERT SPACES

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Abstract. In this paper, we introduce almost expansive curves defined on noncommutative semigroups with the values in a Hilbert space and give an ergodic theorem for such curves. Using this result, we obtain ergodic theorems for almost expansive curves on nonnegative integers and nonnegative real numbers.

Keywords. Almost expansive curve, Banach limit, nonlinear ergodic theorem, expansive mapping, invariant mean.

1 Introduction

In 1975, Baillon [1] originally studied the first nonlinear ergodic theorem in the framework of Hilbert spaces: Let C be a closed and convex subset of a Hilbert space H and let T be a nonexpansive mapping of C into itself. If the set $F(T)$ of fixed points of T is nonempty, then for each $x \in C$, the Cesàro means

$$S_n(x) = \frac{1}{n} \sum_{k=0}^{n-1} T^k x$$

converge weakly to some $y \in F(T)$. In this case, putting $y = Px$ for each $x \in C$, P is a nonexpansive retraction of C onto $F(T)$ such that $PT = TP = P$ and Px is contained in the closure of convex hull of $\{T^n x : n = 1, 2, \dots\}$ for each $x \in C$. We call such a retraction "an ergodic retraction". Takahashi [11] proved the existence of such a retraction for an amenable semigroup of nonexpansive mappings on a Hilbert space. Rodé [9] also found a sequence of means on a semigroup, generalizing the Cesàro means, and extended Baillon's theorem. These results were extended to a uniformly convex Banach space whose norm is Fréchet differentiable in the case of a commutative semigroup of nonexpansive mappings by Hirano, Kido and Takahashi [5]. Recently, in [6], Lau, Shioji and Takahashi extended Takahashi's result and Rodé's result to an amenable semigroup of nonexpansive mappings in the Banach space.

On the other hand, we know Nirenberg's problem [8] concerning the surjectivity of expansive mappings of a Hilbert space H. Let D be a subset of