# A Comparison Result and Lyapunov Stability Criteria with Initial Time Difference

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**Abstract.** We investigate the qualitative behaviour of a perturbed differential system that differs in initial position and initial time with respect to the unperturbed differential system. We compare the classical notion of stability to the notion of initial time difference stability. We present a comparison result which again gives the null solution a central role in the comparison differential system when establishing initial time difference stability of the perturbed differential system with respect to the unperturbed system.
**Key Words:** perturbed differential systems, initial time difference stability, null solution.
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## 1  Introduction

Lyapunov's second method is a standard technique used in the study of the qualitative behavior of differential systems along with a comparison result [2] that allows the prediction of behavior of a differential system when the behavior of the null solution of a comparison system is known. However; there has been difficulty with this approach when trying to apply it to unperturbed differential systems and associated perturbed differential systems with an initial time difference. The difficulty arises because there is a significant difference between initial time difference (ITD) stability and the classical notion of stability. The classical notions of stability are with respect to the null solution, but ITD stability is with respect to the unperturbed differential system where the perturbed differential system and the unperturbed differential system differ both in initial position and initial time [ 3,4,5].

In this paper, we have resolved this difficulty and have a new comparison result which again gives the null solution a central role in the comparison differential system. This result creates many paths for continuing research by direct application and generalization.

In Section 2, we present definitions and necessary background material. In Section 3, we discuss and compare the differences between the classical notion of stability and the recent notion of initial time difference (ITD) stability. In Section 4, we have a comparison result in which the stability properties of the null solution of the comparison system imply the corresponding ITD stability properties of the perturbed differential system with respect to the unperturbed differential system. And in Section 5, we give an example that applies the main result.

## 2  Preliminaries

Consider the differential systems

$$x' = f(t, x), \ x(t_0) = x_0 \ \text{for} \ \ t \geq t_0 \geq 0, \ t_0 \in \mathbb{R}_+ \qquad (2.1)$$

$$x' = f(t, x), \ x(\tau_0) = y_0 \ \text{for} \ \ t \geq \tau_0 \geq 0 \qquad (2.2)$$